Worcester County Mathematics League

WOCOMAL Varsity Meet #4

Coaches' Booklet

March 30, 2005

March 30, 2005 **WOCOMAL Varsity Meet**

Round 1: Elementary Number Theory (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. What is the smallest positive integer, which cannot occur as the difference between two positive prime numbers?

2. If 25_b represents a two-digit number in base b, and if 52_b is twice 25_b, then what is the value of b?

3. What is the value of the positive integer n for which the least common multiple of 36 and n is 500 greater than the greatest common divisor of 36 and n?

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Auburn, Shepherd Hill, Quaboag

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Round 2: Algebra 1 (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Given that M, A, T, and H are positive integers where $9T = 2H$, $5M = 3A$, and $10H = 9A$. Arrange M, A, T, and H in order from least to greatest.

2. If $x + y = 11$ and $y = 15/x$, find the value of $x^2 + y^2$.

3. A girl goes up a ski lift at 4 mph, and comes down the ski slope at 24 mph. If the ski slope is the same length as the ski lift, and you ignore any time spent at the top, her average speed for the round trip, in miles per hour is a/b where a and b are relatively prime. What is the value of (a+b)?

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Tahanto, Worcester Academy, Bromfield

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Round 3: Geometry (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. For how many integer values of x does there exist a triangle whose sides have length $2\frac{1}{2}$, 5, and x ?

2. An equilateral triangle and a regular hexagon have equal perimeters. What is the area of the triangle, if the area of the hexagon is 120?

3. The sides of a triangle are k, k - 3, and k + 3. If the area of the triangle is $\sqrt{1200}$, find the value of k.

ANSWERS

(1 pt.) 1. ________________

(2 pts.) 2. ________________

(3 pts.) 3. ________________

Southbridge, Bancroft, Leicester

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Round 4: Logarithms, Exponents, and Radicals (NO CALCULATORS)

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM.

1. Solve for x if $\log_2(\log_5(\log_3 x) = 1$

2. If the real solution of $x = \sqrt{(x-1/x)} + \sqrt{(1-1/x)}$ is written in the form of $(a + \sqrt{b})/c$, then determine the value of (a+b+c).

3. If $3^{x+2} = 2^{2x-1}$, and $x = \frac{\log A}{\log B}$, then AB = ?

ANSWERS

(1 pt.) 1. ________________

(2 pts.) 2. ________________

(3 pts.) 3. ________________

West Boylston, Westborough, St. John's

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Round 5: Trigonometry (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. For some fixed constant(s) b, the statement $\sin x = \cos(x + b)$ is an identity (true for all x). Find all possible values of b in radians, $0 \le b \le 2\pi$.

2. A triangle has three angles A, B, and C. $\tan A = 1$, $\tan B = x$. Compute $\tan C$ in terms of x.

3. ABCD is a trapezoid with DC parallel to AB, $\angle DCB$ is a right angle, DC = 6, BC = 4, AB = y > 6, and $\angle ADB = x$. If $y = \dfrac{A\sin(x)}{B\cos(x) + C\sin(x)}$ where A,B, and C are all positive integers, then find the value of (A+B+C).

ANSWERS

(1 pt.) 1. _______________

(2 pts.) 2. _______________

(3 pts.) 3. _______________

Doherty, Bromfield

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TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET. (2 points each)

1. When 270 is divided by the odd number n, the quotient is a positive prime number and the remainder is zero. What is n?

2. The quadratic equation $x^2 + cx + d = 0$ has nonzero coefficients c and d. The roots of the equation are also c and d. Find the value of $(3c + d)$.

3. Concentric circles of radius 7 and 11 are drawn in a plane. A chord of the larger circle is trisected at its points of intersection with the smaller circle. What is the length of the chord?

4. What are the real roots of the equation: $x^2 + 18x + 30 = 2\sqrt{(x^2 + 18x + 45)}$?

5. If $\sin x + \sin^2 x + \sin^3 x + \ldots = 4$, then what is the larger of the two possible values for $\cos x + \cos^2 x + \cos^3 x + \ldots$? Write your answer as a fraction in the form of a/b where a and b are relatively prime.

6. Seven white socks and four black socks are in a bag. Two socks are drawn at random, without replacement. What is the probability that they have the same color. Write your answer as a reduced fraction.

7. The number n has 2002 digits, all of which are 2. What is the greatest common divisor of n and 1111?

8. If $2f(x) + f(1-x) = x^2$ for all x, then f(x) = ?

9. In convex quadrilateral HMGS, the diagonals HG and MS meet at point O and the measure of angle HOM is 30^0. Given that the area of ΔHOM is 1, Δ MOG is 2, ΔGOS is 8, and ΔSOH is 4, what is the product of the lengths of the diagonals (HG)(MS)?

Clinton, Bancroft, Notre Dame, Shrewsbury, St. John's

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ANSWERS

Round 1: Elementary Number Theory

1.	(1 pt.)	7
2.	(2 pts.)	8
3.	(3 pts.)	56

Round 2: Algebra 1-open

1.	(1 pt.)	TMHA
2.	(2 pts.)	91
3.	(3 pts.)	55

Round 3: Geometry (OPEN)

1.	(1 pt.)	5
2.	(2 pts.)	80
3.	(3 pts.)	10

Round 4: Logarithms, Exponents, and Radicals

1.	(1 pt.)	$x = 3^{25}$
2.	(2 pts.)	8
3.	(3 pts.)	24

Round 5: Trigonometry (OPEN)

1.	(1 pt.)	$3\pi/2$
2.	(2 pts.)	$(x+1)/(x-1)$
3.	(3 pts.)	31

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TEAM ROUND (2 pts. Each)

1. **135**

2. **1**

3. **18**

4. **$-9 \pm \sqrt{(61)}$**

5. **3/2**

6. **27/55**

7. **11**

8. **$(x^2 + 2x - 1)/3$**

9. **60**

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TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. ____________________________

2. ____________________________

3. ____________________________

4. ____________________________

5. ____________________________

6. ____________________________

7. ____________________________

8. ____________________________

9. ____________________________

March 30, 2005 **WOCOMAL Varsity Meet**

TEAM ROUND **School: ________________________**

Team #:________________________

Team Members:

1. ______________________________________
2. ______________________________________
3. ______________________________________
4. ______________________________________
5. ______________________________________

Total Points for Team Round: ________________________